

DISTORTION OF TRANSIENT SIGNALS ON MULTILAYER COUPLED  
SYMMETRIC MICROSTRIP TRANSMISSION LINES

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## ABSTRACT

Pulse distortion on symmetric coupled microstrips is investigated for multi-layer structures using a simplified Spectral-Domain Approach (SDA). Results for pulse distortion on multi-layer coupled microstrips are shown. Control of the electrical properties of the substrate layers to reduce coupling and cross-talk between adjacent lines is discussed and results are presented.

## I. INTRODUCTION

Ultrafast switching speeds and decreasing circuit dimensions of MMIC's emphasize the dispersive nature of these printed circuit transmission lines. Signal distortion and the close proximity of additional transmission lines necessitate an accurate time domain analysis of coupling between these lines. The distortion of non-periodic signals on single microstrip lines has been investigated using both approximate [1]-[4] as well as exact [5]-[6] formulations and experimental results have also been published [4],[7]. Distortion of signals on multiple strips has also been examined using an impedance matrix approach [8]. However, a full wave analysis of pulse distortion on multiple microstrip lines including the response on adjacent lines has not been investigated. In addition, the reduction of coupling and cross-talk between lines through the control of the electrical parameters of the substrate layers has not been considered.

This paper uses a variation of the Spectral Domain Approach (SDA) to derive the Green's function of the microstrip. The formulation shows that the boundary conditions can be enforced on the  $TE^y$  and  $TM^y$  modes independently, allowing the total boundary-value problem to be split into two smaller, similar problems, thus greatly reducing the complexity of the derivation. Pulse dispersion on coupled planar structures is considered using an even/odd mode formulation. Finally, a method to control coupling in microstrip structures is presented.

## II. DISPERSION RELATIONS

The SDA begins by Fourier transforming the

This work was supported by U.S. Army Research Office under Research Contract DAAG-29-85-K-0078.

fields in the  $x$  and  $z$  directions (Fig. 1). After the fields are transformed, the particular boundary value problem can be solved independently for the  $TE^y$  and  $TM^y$  modes separately. It can be shown by using the continuity equation for the  $TE^y$  modes and the magnetic wave equation for the  $TM^y$  modes that the  $x$  and  $z$  modal currents of each mode are related by

$$\tilde{J}_{xz}^{TE} = -\beta_z \tilde{J}_{zz}^{TE} \quad (1a)$$

$$\beta_z \tilde{J}_{xz}^{TM} = \beta_x \tilde{J}_{zz}^{TM} \quad (1b)$$

Using (1a) and (1b) and that the total currents are the sum of the modal currents gives

$$\tilde{J}_z^{TM} = \beta_z (\tilde{J}_z \beta_z + \tilde{J}_x \beta_x) / (\beta_x^2 + \beta_z^2) \quad (2a)$$

$$\tilde{J}_z^{TE} = \beta_x (\tilde{J}_z \beta_z + \tilde{J}_x \beta_x) / (\beta_x^2 + \beta_z^2) \quad (2b)$$

The modal currents can then be replaced by the total currents in the equations for the electric fields along the interface that contains the conductors. The total fields at the dielectric interface can then be written as:

$$\tilde{E}_z = \frac{j}{\omega \epsilon_0} \left[ \tilde{J}_z \tilde{G}_{zz} + \tilde{J}_x \tilde{G}_{xz} \right] \quad (3a)$$

$$\tilde{E}_x = \frac{j}{\omega \epsilon_0} \left[ \tilde{J}_z \tilde{G}_{zx} + \tilde{J}_x \tilde{G}_{xx} \right] \quad (3b)$$

where  $\tilde{G}_{xx}$ ,  $\tilde{G}_{zx}$ ,  $\tilde{G}_{xz}$  and  $\tilde{G}_{zz}$  are the dyadic Green's functions for the particular geometry being considered and are defined as

$$\tilde{G}_{zz} = \frac{\beta_z^2 \tilde{Z}^{TM} - \beta_x^2 \beta_z^2 \tilde{Z}^{TE}}{\beta_x^2 + \beta_z^2} \quad (4a)$$

$$\tilde{G}_{xx} = \frac{\beta_x^2 \tilde{Z}^{TM} - \beta_z^2 \beta_x^2 \tilde{Z}^{TE}}{\beta_x^2 + \beta_z^2} \quad (4b)$$

$$\tilde{G}_{xz} = \tilde{G}_{zx} = \beta_x \beta_z \frac{\tilde{Z}^{TM} + \beta_0^2 \tilde{Z}^{TE}}{\beta_x^2 + \beta_z^2} \quad (4c)$$

where  $\tilde{Z}^{TE}$  and  $\tilde{Z}^{TM}$  are the input impedances for each mode. Equations (3a-3b) can be solved using Galerkin's method as in [10].

Using  $TE^y$  and  $TM^y$  modes to solve for the Green's function of a multi-layer structure it is possible to represent the function by a simple recurrence formula. For a microstrip structure as in Fig. 1,  $\tilde{Z}^{TM}$  and  $\tilde{Z}^{TE}$  can be written as:

$$\tilde{Z}^{(i)} = \frac{1}{\tilde{Y}_{LN}^{(i)} + \tilde{Y}_{UM}^{(i)}} \quad (3)$$

where  $i$  is either TE or TM, L and U indicate the lower and upper layers, respectively, and  $N$  and  $M$  are the total number of planar layers below and above the interface, as in Fig. 1.  $\tilde{Y}_{(j)}^{(i)}$  can be found using a recursive formulation for either the upper or lower layers, beginning with  $j = L1$  or  $U1$  through  $j = LN$  or  $UM$ , where

$$\tilde{Y}_{(j)}^{TM} = \frac{\tilde{Y}_{s(j)}^{TM} \tilde{Y}_{(j-1)}^{TM} + \alpha_{y(j)}^2 / \mu_{r(j)}^2}{\tilde{Y}_{s(j)}^{TM} + \tilde{Y}_{(j-1)}^{TM}} \quad (4a)$$

$$\tilde{Y}_{(j)}^{TE} = \frac{\tilde{Y}_{s(j)}^{TE} \tilde{Y}_{(j-1)}^{TE} + \alpha_{y(j)}^2 / \epsilon_{r(j)}^2}{\tilde{Y}_{s(j)}^{TE} + \tilde{Y}_{(j-1)}^{TE}} \quad (4b)$$

$$\tilde{Y}_{s(j)}^{TE} = \frac{\alpha_{y(j)} \coth[\alpha_{y(j)} h_{(j)}]}{\mu_{r(j)}} \quad (4c)$$

$$\tilde{Y}_{s(j)}^{TM} = \frac{\epsilon_{r(j)} \coth[\alpha_{y(j)} h_{(j)}]}{\alpha_{y(j)}} \quad (4d)$$

$\tilde{Y}_{(j)}^{(i)}$  can be thought of as the admittance seen looking outward from the  $j^{\text{th}}$  layer (away from the conductor interface) and  $\tilde{Y}_{s(j)}^{(i)}$  is the self admittance of the layer for the particular mode configuration.

### III. PULSE DISTORTION

To compute the pulse dispersion, a Fourier transform approach with an even/odd mode formulation is used. If the propagation constants for the even and odd modes are  $\beta_e$  and  $\beta_o$ , respectively, then the voltage response at a position  $z$  and a time  $t_0$  to an input signal whose Fourier transform is  $V$  can be expressed as;

$$v_1 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{V} \cos\left[\frac{\beta_e - \beta_o}{2}\right] \cos\left[\omega t_0 - z \frac{\beta_e + \beta_o}{2}\right] d\omega \quad (5a)$$

$$v_2 = -\frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{V} \sin\left[\frac{\beta_e - \beta_o}{2}\right] \sin\left[\omega t_0 - z \frac{\beta_e + \beta_o}{2}\right] d\omega \quad (5b)$$

where  $v_1$  is the response on the main line (signal line) and  $v_2$  is the response on the line next to it (sense line).

### IV. NUMERICAL RESULTS

To illustrate signal distortion due to dispersion and coupling, Gaussian pulses of amplitude  $A$  were used, whose half width, half maximum is designated as  $\tau$ . Four different cases are considered in the graphs; the response on the signal line, the response on the sense line, the response if there were no coupling to the sense line (isolated line), and the response if there is neither coupling nor dispersion (undistorted). The difference between the undistorted and the isolated curves shows the effect of dispersion on the pulse,

while the difference between the isolated and the signal line graphs indicate how much affect coupling has on the distortion of the signal. In addition, the sense line response is a measure of the amount of crosstalk in the circuit.

In Fig. 2, a Gaussian pulse ( $\tau = 50$  ps,  $A = 5$ ) travels in a matched microstrip line of dimensions shown, and the response is measured at two different distances. At 40 mm, the pulse has not suffered much distortion from either dispersion or coupling. On the sense line, a response is beginning to form, due to the differences in the even/odd mode  $\epsilon_{\text{eff}}$ . When the pulse reaches 160 mm, however, coupling has severely degraded the signal. Dispersion accounts for a 12 percent reduction in amplitude at this distance, whereas coupling adds an additional 30 percent reduction. In addition, the sense line response has risen to 56 percent of the undistorted signal, creating a spurious response.

The effect of strip spacing on pulse distortion is shown in Fig. 3 for a complex planar microstrip structure. The recurrence formulation was used to compute the Green's function for the structure shown in the figure. As strip spacing is decreased, the amplitude on the signal line is decreased, the pulse is spread out, and the response on the signal line is increased.

While the odd mode  $\epsilon_{\text{eff}}$  of single layer structures is always lower than that of the even mode, for multilayer structures this is not always the case. Fig. 4a shows a structure with a fixed total height but with varying height of the lower dielectric. At two different height ratios, the even and odd modes have the same  $\epsilon_{\text{eff}}$ , meaning that there is no coupling between the strips. The location of these zero coupling points is plotted versus frequency in Fig. 4b. Using this information, it is possible to design microstrip structures which have little or no coupling between lines which are closely spaced. The result of this design is shown in Fig. 5, where the pulses from Fig. 2 have been used again on a structure with the same total height, strip spacing, line width and dielectric constant. The response of the isolated line is not included in this graph, because it is essentially the same as the signal line response. Thus the addition of the layer with a lower dielectric constant has reduced the dispersion slightly and completely eliminated coupling distortion.

### V. CONCLUSIONS

Using a variation of the Spectral Domain Technique, a recursion relation for general planar microstrip structures was derived that is simple to formulate and computationally efficient. Using this formulation and the Fourier transform, signal distortion for both simple and complex multi-layer, multi-conductor lines was investigated as a function of distance and strip spacing. A method for eliminating coupling distortion between multiple lines was discussed, and results were presented that confirmed the removal of coupling despite the close spacing of the lines.

## VI. ACKNOWLEDGEMENT

The authors would like to thank Dr. James W. Mink of the Electronics Division, Army Research Office, for his interest and support of the project and Leesa M. Burns for her constant encouragement and support.

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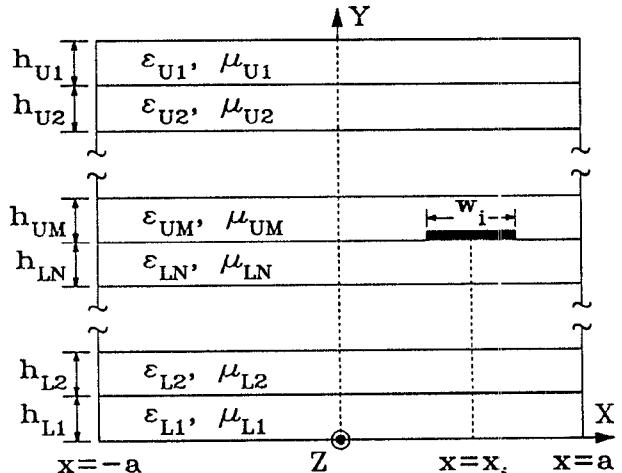


Fig. 1. Multi-layer geometry

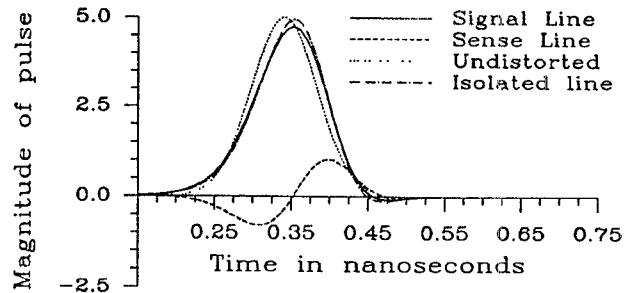


Fig. 2a. Distortion of a Gaussian pulse on coupled lines,  $l=40\text{mm}$ ,  $\tau=50\text{ps}$  for the structure shown below

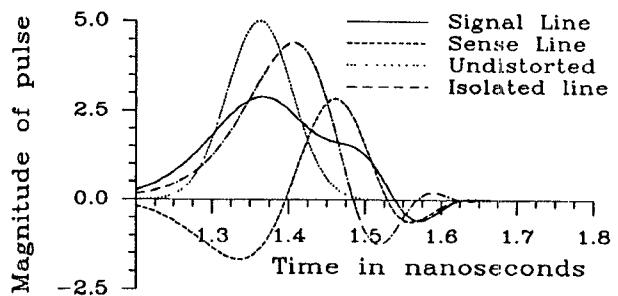
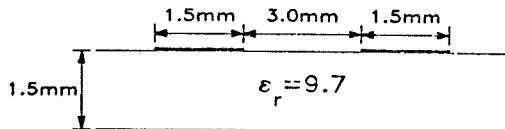
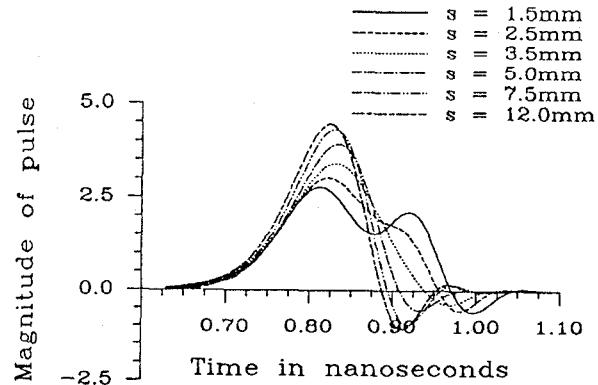
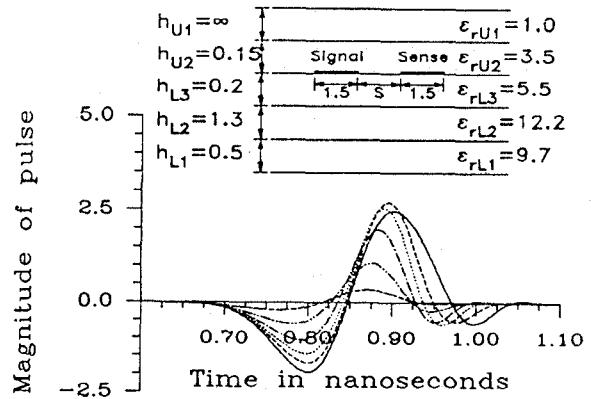


Fig. 2b. Distortion of a Gaussian pulse on coupled lines,  $l=160\text{mm}$ ,  $\tau=50\text{ps}$  for the structure shown above



(a) Signal line



(b) Sense line

Fig. 3. Pulse distortion on multi-layer coupled lines vs. spacing with  $l=100$ mm,  $\tau=50$ ps, for the structure shown above, all dimensions in millimeters

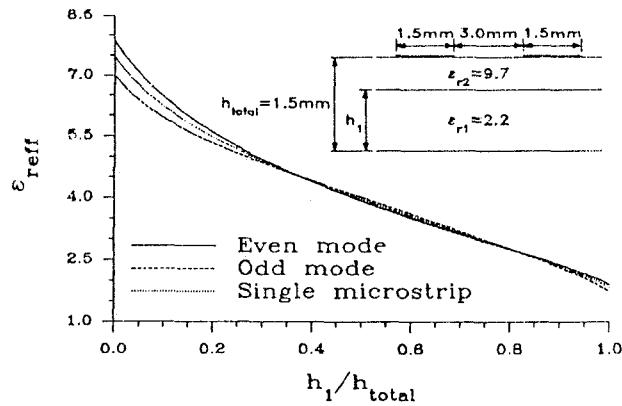


Fig. 4a.  $\epsilon_{\text{eff}}$  vs. height ratio of dielectric substrates  $f=10$ GHz, for the structure shown above.

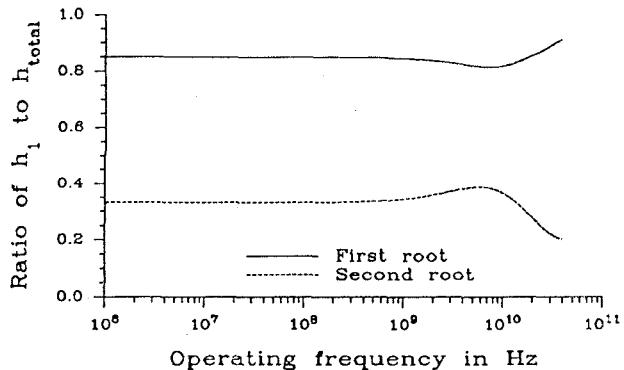


Fig. 4b. Location of zero distortion points vs. frequency for structure in Fig. 4a.

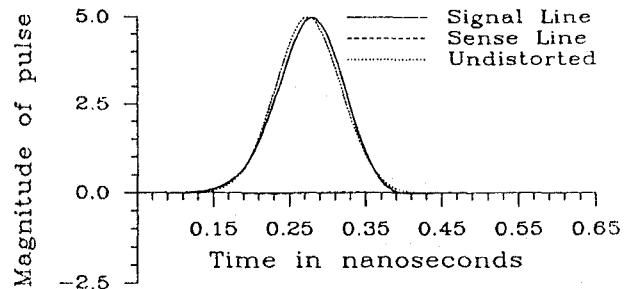


Fig. 5a. Distortion of a Gaussian pulse on coupled lines,  $l=40$ mm,  $\tau=50$ ps for the structure shown below

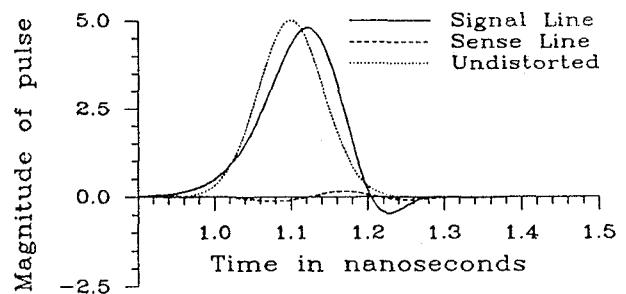
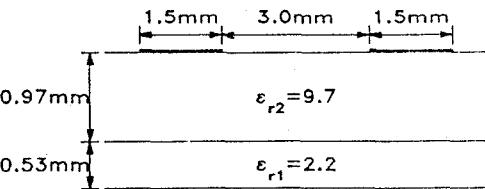


Fig. 5b. Distortion of a Gaussian pulse on coupled lines,  $l=160$ mm,  $\tau=50$ ps for the structure shown above